

Sampling-Based Bayesian Tracking of Topological Features for Mobile Robot Navigation

Domingo Gallardo, Francisco Escolano, Ramón Rizo,
Otto Colomina, Miguel A. Cazorla

Grupo de Visión, Gráficos e Inteligencia Artificial
Departamento de Ciencia de la Computación e Inteligencia Artificial
Universidad de Alicante, Spain

Abstract—Reliable extraction and tracking of environmental structure from noisy range measurements is a fundamental requirement for mobile robot navigation. Traditional approaches usually extract local geometric primitives such as segments or corners from individual sensor scans. These features are often unstable in cluttered and dynamic environments and do not exploit the temporal continuity introduced by robot motion.

This paper presents a Bayesian approach for the estimation and temporal tracking of higher-level environmental features, such as corridors and corridor ends, from ultrasonic range measurements. The posterior density over the continuous parameter space defining each feature is represented explicitly by a set of weighted samples. The samples are propagated according to a stochastic motion model and reweighted according to the likelihood of the current sonar observations. This provides a direct application of the CONDENSATION framework, originally developed for visual tracking, to mobile robot perception.

Unlike Kalman-filter approaches, the proposed method does not require Gaussian posterior densities, and unlike discrete Bayesian methods it does not require an explicit discretization of the parameter space. The resulting representation can therefore maintain several simultaneous hypotheses and support recovery from temporarily missing or corrupted observations, as illustrated in the reported examples.

Experiments with simulated environments containing obstacles and door openings, as well as experiments using sonar measurements from a RWI B-21 mobile robot, show that corridor-level features can be initialized and tracked robustly despite severe variations in the sensor data.

Index Terms—mobile robots, Bayesian estimation, particle filters, CONDENSATION, sonar sensing, feature tracking, probabilistic robotics.

I. INTRODUCTION

Autonomous navigation in indoor environments requires a mobile robot to obtain stable information about the structure of its surroundings. Such information is required for localization, map building, path planning, and the selection of local navigation behaviors.

Ultrasonic range sensors have traditionally been used in indoor mobile robots because of their low cost, low computational requirements, and fast response. However, their measurements are affected by substantial uncertainty. Beam width, surface orientation, multiple reflections, missing echoes, and unmodeled

obstacles may cause measurements to differ considerably from the actual distance to the nearest surface.

Most approaches to sonar interpretation extract elementary geometric primitives, such as line segments, edges, or corners [1], [3], [12]. Although useful, these primitives are local descriptions of the environment and may be highly unstable. A small change in robot position or in the configuration of the environment can cause an individual primitive to disappear or to be replaced by a different one.

In this work we consider a different level of representation. Rather than detecting isolated geometric elements, we estimate persistent environmental structures such as **corridors and corridor ends**. These structures are defined by a small set of continuous parameters and are maintained over time as the robot moves.

The central problem then becomes one of temporal Bayesian estimation. Let x_t denote the parameters defining a feature at time t , and let z_t denote the sonar observations. Instead of computing a single best estimate of x_t , we seek the complete posterior density

$$p(x_t \mid z_{1:t}, a_{1:t-1}), \quad (1)$$

where a_t denotes the motion performed by the robot.

This density may be highly non-Gaussian and, in ambiguous environments, multimodal. A single Gaussian approximation may therefore be inadequate in the situations of interest. A discrete-state Bayesian representation can represent arbitrary densities, but requires the parameter space to be discretized, with a computational cost that increases rapidly with dimensionality.

Our approach is to represent the posterior density by a finite set of weighted samples. We use the CONDENSATION algorithm introduced by Isard and Blake [9] for stochastic visual contour tracking. In our application, however, the samples do not represent image contours. They represent hypotheses about geometric structures surrounding the robot.

The main contributions of this paper are:

1. the formulation of corridor-level environmental perception as temporal Bayesian estimation over a continuous parameter space;
2. the application of sample-based posterior propagation to the tracking of such features;

3. the integration of robot motion and sonar observations in a single probabilistic framework;
4. experimental evidence showing that the representation remains stable in the presence of unmodeled obstacles, missing measurements, doors, and highly noisy real sonar data.

The remainder of the paper is organized as follows. Section II presents the Bayesian formulation. Section III describes the sampling algorithm. Section IV defines the feature, motion, and observation models. Section V presents experimental results. Finally, Sections VI and VII discuss the implications and summarize the main conclusions.

II. TEMPORAL BAYESIAN ESTIMATION OF ENVIRONMENTAL FEATURES

A. Feature estimation

Consider an environmental feature described by a continuous parameter vector

$$x = (x_1, \dots, x_n). \quad (2)$$

For example, a corridor may be represented by its orientation relative to the robot and the distances from the robot to its two walls. A corridor end additionally requires the distance to the closing wall.

Given a set of sensor observations z , Bayesian estimation provides the posterior density

$$p(x | z) = \alpha p(z | x) p(x), \quad (3)$$

where $p(x)$ is the prior density, $p(z | x)$ is the observation likelihood, and α is a normalization constant.

The posterior represents all available information concerning the feature. In particular, it is not necessary to commit prematurely to a single parameter estimate.

This is especially important for sonar perception. Different environmental configurations may generate similar measurements, producing several separated regions of high probability.

B. Temporal estimation

When the robot moves, the feature parameters expressed in robot-centered coordinates evolve over time.

Let

$$Z_t = \{z_1, \dots, z_t\} \quad (4)$$

be the sequence of observations and

$$A_{t-1} = \{a_1, \dots, a_{t-1}\} \quad (5)$$

the sequence of robot motions.

We assume a first-order Markov model,

$$p(x_t | x_{1:t-1}, A_{t-1}) = p(x_t | x_{t-1}, a_{t-1}), \quad (6)$$

and that the current observation depends only on the current feature state,

$$p(z_t | x_{1:t}, Z_{t-1}, A_{t-1}) = p(z_t | x_t). \quad (7)$$

The recursive Bayesian estimator consists of two stages.

1) *Prediction*: The posterior from the previous iteration is propagated through the motion model:

$$p(x_t | Z_{t-1}, A_{t-1}) = \int p(x_t | x_{t-1}, a_{t-1}) \times p(x_{t-1} | Z_{t-1}, A_{t-2}) dx_{t-1}. \quad (8)$$

2) *Observation update*: The prediction is then corrected using the current sonar measurements:

$$p(x_t | Z_t, A_{t-1}) = \alpha_t p(z_t | x_t) p(x_t | Z_{t-1}, A_{t-1}). \quad (9)$$

The remaining question is how to represent and propagate this density efficiently.

III. SAMPLE-BASED REPRESENTATION OF THE POSTERIOR

Kalman filtering represents the posterior through a mean vector and a covariance matrix. This representation is efficient when the relevant densities are approximately Gaussian, but it cannot represent multiple competing interpretations of a sonar scan.

A discrete Bayesian estimator can represent arbitrary distributions by assigning a probability to every state. However, discretization becomes expensive when several continuous parameters must be estimated simultaneously.

We instead approximate the posterior at time t by a set

$$S_t = \left\{ (x_t^{(i)}, w_t^{(i)}) \right\}_{i=1}^N, \quad (10)$$

where each $x_t^{(i)}$ is a feature hypothesis and $w_t^{(i)}$ its associated weight.

The total probability represented by samples located within a region approximates the probability mass of that region.

A. Sampling algorithm

At initialization, if no prior information is available, the samples are distributed uniformly throughout the admissible parameter space.

At each iteration, Algorithm 1 is applied. The previous weights are normalized; the observation update assumes a positive total weight.

This is the stochastic propagation mechanism used by CONDENSATION [9].

The important difference from its original visual-tracking application lies in the interpretation of the state. Here, a sample is a hypothesis about an environmental structure.

Algorithm 1 Sample-based feature tracker

Input: S_{t-1} , robot motion a_{t-1} , observation z_t .**Output:** Weighted posterior approximation S_t .

- 1) For each $i = 1, \dots, N$:
 - a) Select ancestor j with probability $w_{t-1}^{(j)}$.
 - b) Draw $\tilde{x}_t^{(i)} \sim p(x_t | x_{t-1}^{(j)}, a_{t-1})$.
 - c) Compute $\tilde{w}_t^{(i)} = p(z_t | \tilde{x}_t^{(i)})$.
 - 2) Normalize $w_t^{(i)} = \tilde{w}_t^{(i)} / \sum_{k=1}^N \tilde{w}_t^{(k)}$.
 - 3) Set $S_t = \{(\tilde{x}_t^{(i)}, w_t^{(i)})\}_{i=1}^N$.
-

IV. MODELS

A. Corridor representation

We consider two high-level environmental structures.

A corridor is described by three parameters,

$$x_c = (d_l, d_r, \theta), \quad (11)$$

where d_l and d_r are the distances to the two corridor walls and θ is the orientation of the corridor relative to the robot.

A corridor end requires an additional parameter,

$$x_e = (d_l, d_r, d_f, \theta), \quad (12)$$

where d_f represents the distance to the wall closing the corridor.

These features provide a more stable description of indoor structure than independently extracted edges or line segments.

They can also be interpreted directly by higher-level navigation modules. For example, the estimated parameters can be used to center the robot within a corridor or to detect that the robot is approaching its end.

B. Motion model

The motion model used in the original experiments assumes a synchro-drive configuration.

Let its linear and angular velocities be $v(t)$ and $\omega(t)$. Writing the robot position as (r_x, r_y) and its heading as φ to distinguish them from the feature state, its pose evolves according to

$$\varphi(t_n) = \varphi(t_0) + \int_{t_0}^{t_n} \omega(t) dt, \quad (13)$$

$$r_x(t_n) = r_x(t_0) + \int_{t_0}^{t_n} v(t) \cos \varphi(t) dt, \quad (14)$$

$$r_y(t_n) = r_y(t_0) + \int_{t_0}^{t_n} v(t) \sin \varphi(t) dt. \quad (15)$$

For the discrete controller used in our experiments, v and ω are assumed approximately constant during each sampling interval.

The feature parameters are transformed according to the relative motion of the robot. Gaussian perturbations are subsequently added to the transformed parameters in order to represent motion and odometry uncertainty:

$$x_t^{(i)} = f(x_{t-1}^{(i)}, a_{t-1}) + \epsilon, \quad (16)$$

where

$$\epsilon \sim \mathcal{N}(0, \Sigma_a). \quad (17)$$

This stochastic component is important. Without it, repeated resampling would progressively eliminate hypotheses that could later become consistent with the observations.

C. Sonar observation model

The robot obtains a vector of measurements

$$z = (l_1, \dots, l_{24}) \quad (18)$$

from a ring of 24 ultrasonic sensors.

For every hypothesized feature configuration x , an expected sonar observation

$$\hat{z}(x) \quad (19)$$

is generated.

The simulator is based on a two-dimensional adaptation of ray tracing. Besides direct returns, the model is capable of reproducing measurements resulting from indirect reflections. Such measurements are common in ultrasonic sensing and cannot in general be explained by simply adding Gaussian noise to an ideal range measurement.

The resulting likelihood is defined from the discrepancy between the actual and simulated scans:

$$p(z | x) \propto \exp \left[-\frac{d(z, \hat{z}(x))^2}{\sigma^2} \right]. \quad (20)$$

In the experiments presented here, d is computed from the Euclidean discrepancy between the endpoints of the measured and simulated range readings.

This expression specifies the unnormalized likelihood score used by the tracker; σ controls its scale. The original sonar simulation is described in [7]. This procedure follows an analysis-by-synthesis interpretation of perception: a hypothesized environmental configuration is evaluated according to its ability to generate the current observations.

V. EXPERIMENTAL RESULTS

Experiments were designed to investigate four properties of the proposed representation:

1. initialization from a broad prior;
2. robustness to isolated unmodeled obstacles;
3. persistence when the expected structure temporarily disappears from the measurements;
4. behavior with noisy measurements obtained from a real mobile robot.

The simulated experiments below reproduce those reported in [16]; the real-data example comes from [17]. Unless stated otherwise, $N = 300$ samples were used.

The simulated robot moved at approximately 25 cm/s, and sonar observations were obtained every 0.25 s.

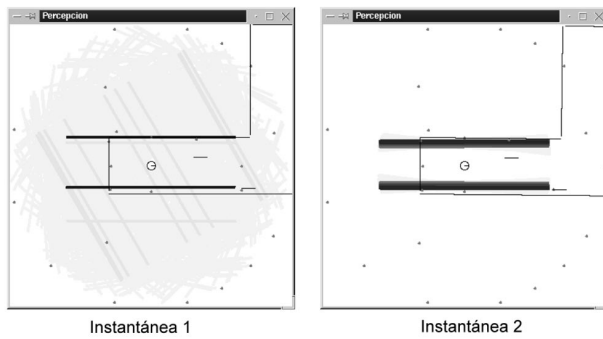


Fig. 1. Initial particle population and posterior concentration during corridor initialization. Original CCIA 1998, Fig. 5 [16]; snapshots are separated by one second.

On a Pentium II processor, the unoptimized implementation required approximately four times the available real-time computation, suggesting that real-time operation should be feasible after implementation optimization.

A. Initialization

The first experiment considers a robot located inside a corridor (Fig. 1).

The initial particle population covers a broad range of possible corridor parameters. After sonar measurements are incorporated, samples representing corridor geometries inconsistent with the observations receive low probability and disappear during resampling.

After successive observations, the distribution concentrates around a small region corresponding to the actual corridor parameters.

This experiment illustrates an important difference with deterministic feature extraction. No explicit decision about corridor parameters needs to be made from a single sonar scan. Uncertainty can instead be preserved until sufficient evidence has accumulated.

B. Tracking in the presence of an obstacle

In the second experiment, the robot moves along a corridor containing an obstacle not represented in the corridor model (Fig. 2).

Before reaching the obstacle, the samples are concentrated around the correct corridor geometry.

When the obstacle enters the sonar field, the likelihood of most existing hypotheses decreases because several measurements no longer correspond to the expected corridor walls.

The stochastic motion model nevertheless maintains a population of alternative hypotheses. As the robot passes the obstacle and observations of the corridor walls become available again, samples corresponding to the actual corridor geometry regain high probability and the distribution reconverges.

The experiment demonstrates that temporary sensor inconsistencies do not necessarily cause the feature estimate to be discarded.

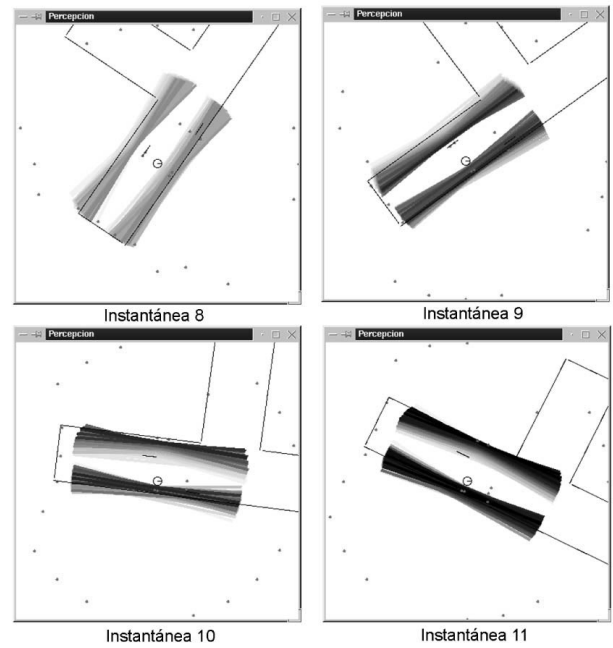


Fig. 2. Corridor tracking past an unmodeled obstacle. Original CCIA 1998, Fig. 6 [16], snapshots 8–11. Darker hypotheses have higher likelihood.

C. Tracking corridor ends through doors

The third experiment (Fig. 3) is deliberately more difficult.

The robot follows a corridor containing several doors while estimating a corridor-end feature.

As the robot passes in front of the openings, the sonar measurements contain little or no direct evidence supporting the expected corridor structure. During this interval, the posterior is propagated mainly according to the motion model and gradually becomes more diffuse.

When the robot again observes the corridor structure, the corresponding hypotheses are reinforced and the posterior rapidly reconcentrates.

This result is significant because it illustrates the temporal nature of the representation. The feature does not have to be independently redetected in every scan. Instead, its probability distribution persists through intervals in which the current observation is weak or ambiguous.

D. Experiments with real sonar data

The following experiment is taken from Section 5.4 of the 1999 thesis [17], rather than the 1998 CCIA paper [16]. The thesis explicitly uses the motion and observation models developed in its Chapter 4. It is therefore evidence for the sampling-based feature-tracking framework with that later model, not a controlled validation of the simple scan-distance likelihood above.

The method was also evaluated using measurements obtained with PIXIE, an RWI B-21 mobile robot equipped with a ring of 24 ultrasonic sensors.

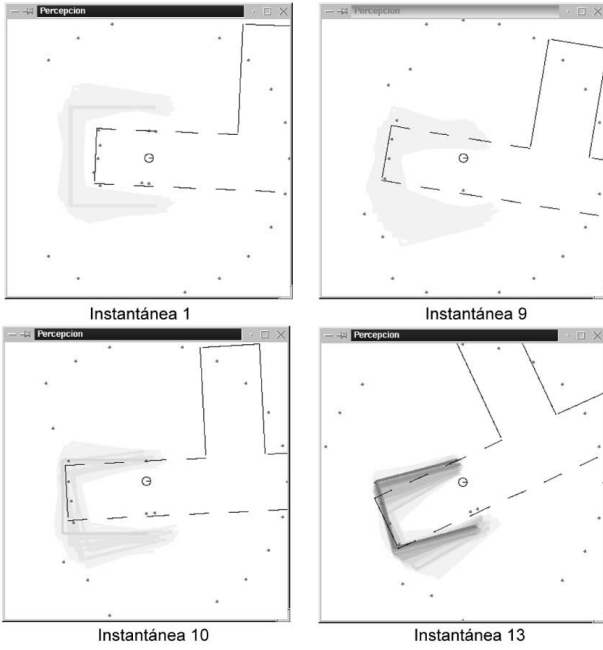


Fig. 3. Corridor-end tracking through door openings. Original CCIA 1998, Fig. 7 [16], snapshots 1, 9, 10, and 13.

The robot moved inside a real corridor at approximately 25 cm/s, with sensor readings acquired every 0.25 s. As in the previous experiments, 300 samples were used.

Figures 4 and 5 show the posterior samples and mean corridor estimates. The measurements were substantially noisier than those generated by the simulator. In particular, columns located along one side of the corridor generated significant disturbances in the range data.

Nevertheless, the particle distribution remained centered around corridor configurations compatible with the actual geometry while the robot moved along the corridor.

The reported snapshots illustrate the applicability of the framework to real sonar observations; no additional numerical accuracy measurements are inferred from these figures.

VI. DISCUSSION

The experimental results show that sample-based Bayesian estimation provides a useful representation for structural mobile-robot perception.

A central advantage is that uncertainty is represented explicitly.

A deterministic feature detector typically produces either a single feature estimate or no feature at all. The proposed method instead maintains a probability distribution over possible feature configurations. Weak observations broaden this distribution rather than immediately destroying the estimate.

This property is particularly useful for ultrasonic sensing.

Doors, furniture, columns, people, and unfavorable reflection geometries frequently cause the environment observed in a single sonar scan to differ substantially from an ideal geometric model. Temporal propagation allows information acquired

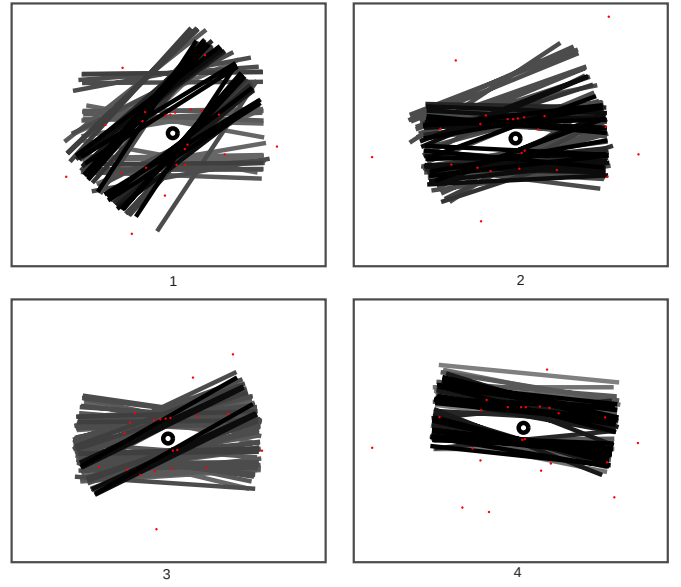


Fig. 4. Four posterior sample distributions using real PIXIE sonar data. Reproduced from the thesis, Fig. 5.7, p. 109 [17].

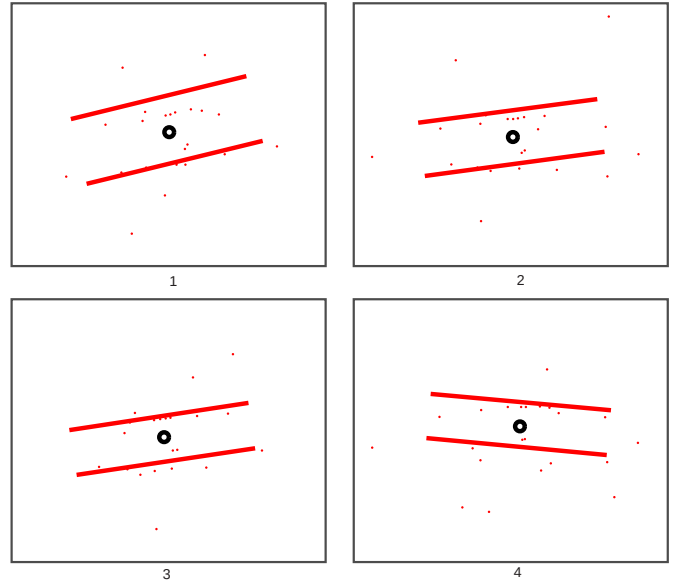


Fig. 5. Mean estimated corridor configurations for the real-data experiment. Points are measured sonar endpoints. Reproduced from the thesis, Fig. 5.8, p. 110 [17].

before and after such disturbances to contribute to the same feature estimate.

The representation also naturally permits multimodal distributions.

Consider, for example, an observation compatible with two different corridor orientations or two geometrically similar structures. A Gaussian estimator would summarize both possibilities by a mean that may correspond to neither valid interpretation. A sample population can instead maintain both hypotheses

until later observations resolve the ambiguity.

There is also a close conceptual relation between the method proposed here and stochastic visual tracking.

CONDENSATION was introduced to maintain multiple hypotheses concerning the state of a moving visual contour [9]. In the present work, the same mechanism is used to track an abstract geometric entity inferred from range measurements.

Thus the algorithm is not specific to images or contours. It can be regarded as a general mechanism for recursively estimating continuous, nonlinear, and non-Gaussian state variables.

The principal computational expense is the repeated synthesis of a sonar scan for each hypothesis. The reported implementation did not operate in real time; optimization remains a requirement, rather than an experimentally demonstrated result.

These examples provide qualitative evidence, not a quantitative comparison with alternative estimators. They do not establish error bounds, success rates, or superiority over Kalman or discrete-state methods. Finite particle sets can also lose plausible modes during resampling. The ability to represent several modes should therefore not be interpreted as a guarantee that every relevant hypothesis will survive.

The present formulation is related to earlier sonar interpretation and mapping work [2], [4], [14] and probabilistic robot navigation [5], [10], [11], [15]. Its specific focus is the continuous state of a local structural feature. Statistical recognition [6], analysis by synthesis [8], and numerical sampling methods [13] provide the broader methodological context.

VII. CONCLUSIONS

We have presented a sampling-based Bayesian formulation for tracking corridor-level environmental features from ultrasonic measurements. A population of weighted samples represents uncertainty over continuous geometric parameters; resampling, stochastic motion propagation, and observation weighting implement the recursive estimator. The resulting approach transfers the CONDENSATION mechanism from visual tracking to structural mobile-robot perception.

The original simulated experiments demonstrate initialization and continued tracking in corridors containing unmodeled obstacles and door openings. The later thesis reports an application of the same estimation framework to real PIXIE sonar measurements, using its more developed observation model. Together, these results support the feasibility of temporal feature estimation while leaving quantitative performance and real-time execution to further evaluation.

Future work includes reducing the cost of sonar simulation, evaluating sensitivity to the number of samples and model parameters, and testing additional environmental structures. Comparisons using repeated trials and explicit geometric error measurements would establish more precisely when the proposed representation is advantageous.

REFERENCES

- [1] B. Barshan and R. Kuc, "Differentiating sonar reflections from corners and planes by employing an intelligent sensor," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 12, no. 6, pp. 560–569, Jun. 1990.
- [2] D. Pagac, E. M. Nebot, and H. Durrant-Whyte, "An evidential approach to probabilistic map-building," in *Proc. IEEE Int. Conf. Robotics and Automation*, 1996.
- [3] M. Drumheller, "Mobile robot localization using sonar," *IEEE Trans. Pattern Anal. Mach. Intell.*, vol. 9, no. 2, pp. 325–332, 1987.
- [4] A. Elfes, "Sonar-based real-world mapping and navigation," *IEEE J. Robotics and Automation*, vol. 3, no. 3, pp. 249–265, Jun. 1987.
- [5] D. Fox, W. Burgard, S. Thrun, and A. B. Cremers, "Position estimation for mobile robots in dynamic environments," in *Proc. 15th Nat. Conf. Artificial Intelligence (AAAI)*, 1998.
- [6] K. Fukunaga, *Introduction to Statistical Pattern Recognition*. Academic Press, 1990.
- [7] D. Gallardo, O. Colomina, and R. Rizo, "Una función de verosimilitud multimodal para las lecturas de sonares en robots móviles," Departamento de Ciencia de la Computación e Inteligencia Artificial, Universidad de Alicante, Tech. Rep., 1998.
- [8] U. Grenander, *General Pattern Theory: A Mathematical Study of Regular Structures*. Oxford University Press, 1993.
- [9] M. Isard and A. Blake, "Contour tracking by stochastic propagation of conditional density," in *Proc. European Conf. Computer Vision (ECCV)*, 1996, pp. 343–356.
- [10] S. Koenig and R. Simmons, "Xavier: A robot navigation architecture based on partially observable Markov decision process models," in *Artificial Intelligence and Mobile Robots*, D. Kortenkamp, R. P. Bonasso, and R. Murphy, Eds. AAAI Press, 1998.
- [11] D. Kortenkamp, R. P. Bonasso, and R. Murphy, Eds., *Artificial Intelligence and Mobile Robots*. AAAI Press, 1998.
- [12] P. J. McKerrow, "Echolocation—from range to outline segments," *Robotics and Autonomous Systems*, vol. 11, pp. 205–211, 1993.
- [13] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical Recipes in C*. Cambridge University Press, 1988.
- [14] A. M. Sabatini and O. Di Benedetto, "Towards a robust methodology for mobile robot localization using sonar," in *Proc. IEEE Int. Conf. Robotics and Automation*, vol. 4, 1994, pp. 3142–3148.
- [15] S. Thrun, "Learning metric-topological maps for indoor mobile robot navigation," *Artificial Intelligence*, vol. 99, no. 1, pp. 21–72, 1998.
- [16] D. Gallardo, F. Escolano, R. Rizo, O. Colomina, and M. A. Cazorla, "Estimación bayesiana de características en robots móviles mediante muestreo de la densidad a posteriori," CCIA, 1998.
- [17] D. Gallardo López, "Aplicación del muestreo bayesiano en robots móviles: Estrategias para localización y estimación de mapas del entorno," Ph.D. dissertation, Departamento de Tecnología Informática y Computación, Universidad de Alicante, May 1999, Sec. 5.4, pp. 102–110.