# Visual Homing Navigation With Two Landmarks: The Balanced Proportional Triangulation Method

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***Abstract*— Current approaches to the visual homing for mobile robot navigation are generally inspired in insects' behavior and based on the observed angular information of fixed points of the environment (landmarks). They suppose the capacity for absolute orientation of the observations (compasses) and/or require the detection and identification of at least three landmarks; furthermore, they tend to generate homing paths with circumvolutions which reduce the efficiency of navigation.**

**This document proposes a new approach to the visual homing navigation based on the observed angular information of only two landmarks, without any previous knowledge of their locations and real sizes or any compasses or range sensors. The bearing towards the home point is calculated by means of triangulation/trilateration, considering the apparent size observed for each landmark and obtaining rectilinear navigation paths.**

**The proposed visual homing navigation method has been simulated widely in an obstacle-free and uncertainty-exempt environment, verifying its robustness and efficiency in the navigation of short and long range with different scene configurations; it can be easily integrated with other navigation systems (e.g.: obstacle avoidance), and even be used to calculate the robot's relative location.**

## I. INTRODUCTION.

The present document proposes a new approach for autonomous robot navigation called "visual homing"; the goal is to reach a given point in the space, which may have been previously visited, by means of the captured visual information of the environment. In the usual mechanisms of visual homing, the robot stores a snapshot of the environment in the returning point (*home*); subsequently, when the robot has to return to that position, compares successively its view of the environment with the stored snapshot, thus generating a motion vector towards the home point according to the discrepancies detected between both views. These discrepancies are determined by means of fixed objects or traits of the environment (*landmarks*) which can be detected and identified as navigation references.

The new homing navigation method proposed does not use any kind of metric maps, compasses or range sensors, and it can reach the whole 2D space - obstacle-free and uncertainty-exempt -. This method considers only two landmarks of the environment mutually distinguishable and identifiable, and it uses the panoramic vision as the only perception system of such environment. Although the coordinates and sizes of the landmarks are unknown, as well as the position and orientation of both the robot and the home point, the proposed method calculates the bearing which must be followed by the robot to reach the home point by means of triangulation/trilateration, as a result of the angular difference observed between the landmarks and of the apparent size of each one in the observations. The accomplished simulations show that through the application of this method the home point is always reached without error.

The term "homing" comes from the field of Biology [15], and describes the ability of various sorts of animals to return to their nests after having gone away from them. Multiple studies on this behavior are available, above all in insects: the "snapshot model" is defined in [4], suggesting that insects store a snapshot of the landmarks obtained in the home point, that they consider the apparent size of such landmarks to reach it, that the snapshots are aligned using a biological compass, and that they get greater efficiency starting from the existence of three landmarks in the scene; the snapshot model limitation to the vicinity of the home point is shown in [6], providing with several approach strategies for long distance navigation. The homing behavior can also be contrasted in several studies about water rats learning: water rats can locate a hidden platform starting from several landmarks of the environment, as is shown in [11]; the direction mechanisms in rodents seem to be related to distant landmarks (they do not have a biological compass as observed in insects), as is suggested in [9]; a minimum of two landmarks are necessary to attain homing, as is shown in [13]; and water rats only consider the nearest two landmarks to the home point, as is suggested in [14].

Inspired in homing behavior observed in insects, several implementation approaches have been carried out in real robots for autonomous navigation: the snapshot model is implemented in [10], using a robot which can orient itself by polarizing the sunlight, and also defining the vectorial motivation strategies "Proportional Vector" and "Average Landmark Vector"; the computational bases of the snapshot model are analyzed in [7], as well as the "Average Displacement Vector" method is proposed and implemented; and a homing navigation method which does not use a compass is proposed in [3]. The existing homing navigation approaches based on insects' behavior are summarized in [8], pointing out that insects either use some kind of compass to guide the observations, and/or need the observation of at least three landmarks in the environment.

Concerning homing behavior observed in water rats, no implementation approach has been found in an autonomous robot. Although the homing navigation method proposed in this document is not inspired in the studies about water rats learning, there is a close parallelism with them.

The proposed method focuses on the following underlying idea: the proportional distances of two objects to the observation point can be deduced from the apparent size of them in two different images; therefore, the relative location is possible by triangulation/trilateration considering the obtained angle-based information from two panoramic images containing only two landmarks - being mutually distinguishable and identifiable -, and without any previous knowledge of their coordinates or sizes. Extrapolating, the proposed method is valid for any sensor of the environment which gives information about direction and "size" of each landmark; as for example, the wireless antennae [12], which permit to discriminate the reception angle of the signal from a transmitter (landmark), considering its intensity of reception as the landmark "size".

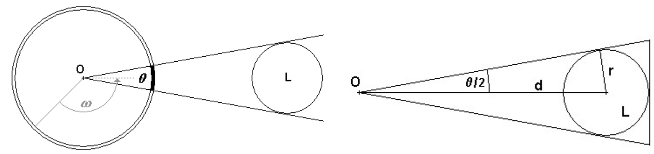
The contribution of this work to visual homing navigation for autonomous robots is double: on the one hand, it demonstrates that it is feasible to carry it out considering only two landmarks of the environment, with no compass use, range sensors, environment metric maps or any express knowledge on the landmark coordinates and sizes; and on the other hand, it demonstrates that it can be calculated by means of triangulation/trilateration, thus obtaining rectilinear navigation paths (without the characteristic circumvolutions of the methods based on the sum of motivation vectors) and resolving implicitly the relative location of the robot.

The rest of the document is structured as follows: section II describes briefly the technique for obtaining panoramic images, as well as the inherent information to landmark observation. Section III details the method proposed for the calculation of the home bearing in each navigation stage by means of balanced proportional triangulation. Section IV introduces the results obtained by testing the method on a navigation simulator built for the purpose, followed by a discussion in section V about several aspects of the proposed method. Finally, the conclusions of the work and the future research lines are shown in section VI.

## II. OBJECT OBSERVATION IN A PANORAMIC IMAGE.

There are a lot of available studies detailing the obtaining, conversion and treatment techniques of panoramic images (e.g.: [5]). Basically, a panoramic image is obtained by means of a conventional optics camera on which a mirror lens is placed. This lens has a parabolic, hyperbolic or spherical shape, and permits to get an omni-directional image of the environment, which is converted into a panoramic image by transforming the polar coordinates into Cartesian ones.

The use of panoramic images simplifies the visual perception of the environment, since an image of the whole environment around the observation position is obtained with a single snapshot.




(a) (b)

Fig. 1. (a) Weighted observation of an object in a panoramic image. (b) Calculation of the distance from the centroid of an object with a known radius, starting from the observation of the object.

### II-A. *Weighted object observation in a panoramic image.*

In a panoramic image, each identifiable object $L$ of the environment occupies a circular sector $\theta$ of the longitudinal plane of the image (Fig. 1(a)), which represents the weight or apparent size of the object in the observation. Furthermore, the bisector of $\theta$ forms an angle $\omega$ related to a fixed and subjective reference system (e.g.: the frontal position of the robot), which permits to angle-based locate the centroid of the observed object in relation to the observation position.

The magnitude of $\theta$ is related both to the distance $d$ between the observation point $O$ and the the centroid of the object $L$, and to its radius $r$ (Fig. 1(b)). If we know the radius $r$ of the observed object, we can determine the distance $d$ as follows:

$$d = \frac{r}{\sin(\theta/2)} \tag{1}$$

### II-B. *Weighted observation of two objects in a panoramic image.*

In a panoramic image, the observation of two objects $L_1$ and $L_2$ permits to determine the angular difference $\alpha \in [0, 2\pi[$ between the centroids of both objects, being calculated by the form: $\alpha = \omega_2 - \omega_1$.

If the radii $r_1$ and $r_2$ of the objects $L_1$ and $L_2$ are known, and considering both $\alpha$ and the corresponding $\theta_i$ and $\omega_i$, it is possible to determine the relative location of the observation point $O$ (Fig. 2): the segments $O_i$ are calculated by applying (1); the segment $b$ is calculated applying the cosine law; and the location of the point $O$ is determined through simple triangulation. The location obtained in this way is relative to the coordinates of the centroids of the observed objects.

## III. HOMING BASED ON THE WEIGHTED OBSERVATION OF TWO LANDMARKS.

Let's consider a scene without any absolute external references, obstacle-free and with two fixed objects (*landmarks*), whose coordinates and sizes will be unknown. Let's also consider a robot which is able to obtain panoramic images of the scene (henceforth, *observations*), uncertainty-exempt and in which both landmarks will always be mutually visible and identifiable.

Using exclusively the angular information coming from the observations[1] (see II-A and II-B), we can calculate the leading

[1]As a convention: all the angles will be considered as positive in a counterclockwise order; the bow of the robot will be denoted as $h$, being considered as subjective direction reference for the panoramic images.

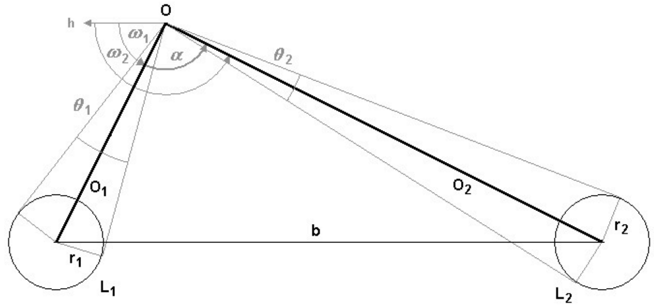



Fig. 2. Weighted observation of two objects in a panoramic image.

relative bearing - through triangulation/trilateration -, from any point $R$ of the scene, in order to reach another point $H$ (home) previously visited, from which an observation of the environment may have been obtained and stored. In Fig. 3 the general calculation diagram is shown; its goal is to determine the angle $R\gamma_1$ which, added to the observation angle $R\omega_1$ of the landmark $L_1$, indicates the bearing of the point $H$ (home) relative to the subjective reference $Rh$ (bow of the robot) of the point $R$ (unknown current pose of the robot). The previous step to the calculation of the triangulation is the determination of the linear distances between the observation points and the centroids of both landmarks, applying (1):

$$\begin{aligned} R_1 &= \tfrac{r_1}{\sin(R\theta_1/2)} \quad ; \quad R_2 = \tfrac{r_2}{\sin(R\theta_2/2)} \\ H_1 &= \tfrac{r_1}{\sin(H\theta_1/2)} \quad ; \quad H_2 = \tfrac{r_2}{\sin(H\theta_2/2)} \end{aligned} \tag{2}$$

Once these distances have been determined, the leading relative bearing so as to reach the point $H$ from the point $R$ is obtained through the resolution of the following triangulation equations, where the *sign(x)* function returns the value -1 for all $x < 0$, and 1 for another case:

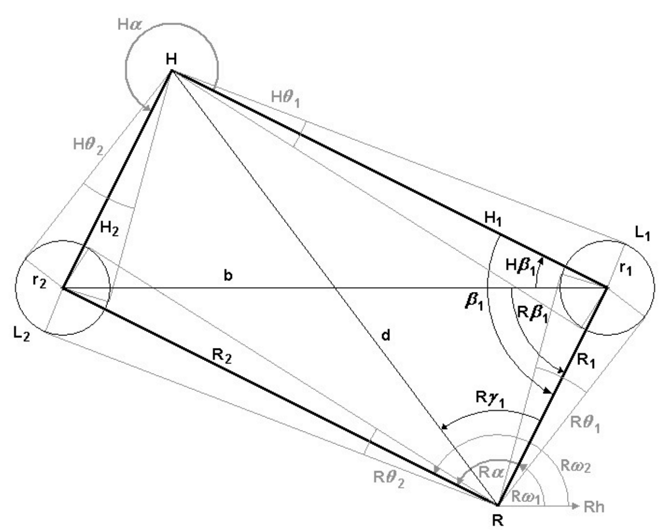



Fig. 3. General calculation diagram of the home bearing by means of triangulation / trilateration, starting from two observations ($H$ and $R$) of two fixed objects of the environment ($L_1$ and $L_2$).

$$\begin{aligned} & b = \sqrt{R_1^2 + R_2^2 - 2 \cdot R_1 \cdot R_2 \cdot \cos R\alpha} \quad ; \\ & R\beta_1 = \arccos\left(\tfrac{b^2+R_1^2-R_2^2}{2 \cdot b \cdot R_1}\right) \cdot sign(\sin R\alpha) \quad ; \\ & H\beta_1 = \arccos\left(\tfrac{b^2+H_1^2-H_2^2}{2 \cdot b \cdot H_1}\right) \cdot sign(\sin H\alpha) \quad ; \\ & \beta_1 = R\beta_1 - H\beta_1 \quad ; \\ & d = \sqrt{R_1^2 + H_1^2 - 2 \cdot R_1 \cdot H_1 \cdot \cos \beta_1} \quad ; \\ & R\gamma_1 = \arccos\left(\tfrac{d^2+R_1^2-H_1^2}{2 \cdot d \cdot R_1}\right) \cdot sign(\sin \beta_1) \quad ; \\ & bearing = R\omega_1 + R\gamma_1 \end{aligned} \tag{3}$$

The triangulation method proposed up to now is based on the knowledge of the radii $r_1$ and $r_2$ of the two observed landmarks. However, this information is not available.

### *III-A. Home bearing calculation through proportional triangulation.*

If the radii of the observed landmarks are unknown, both radii can be supposed to possess the same magnitude (e.g., the value 1). Under such supposition, the calculation of the linear distances between the observation points and the centroids of both landmarks is accomplished by applying (1) as follows:

$$\begin{aligned} R_1' &= \tfrac{1}{\sin(R\theta_1/2)} \quad ; \quad R_2' = \tfrac{1}{\sin(R\theta_2/2)} \\ H_1' &= \tfrac{1}{\sin(H\theta_1/2)} \quad ; \quad H_2' = \tfrac{1}{\sin(H\theta_2/2)} \end{aligned} \tag{4}$$

This way we obtain a triangulation which is proportional and equivalent to the real (Fig. 4(a)), and enough to determine the relative bearing to the point $H$ from the point $R$, through the resolution of the triangulation (3), and using $R_1'$, $R_2'$, $H_1'$ and $H_2'$ instead of $R_1$, $R_2$, $H_1$ and $H_2$, respectively.

The proportional triangulation method proposed up to now is based on the supposition of the fact that the radii of the two landmarks observed ($r_1$ and $r_2$) are equal. If this is not certain, the obtained relative bearing will be wrong.

### *III-B. Home bearing calculation through balanced proportional triangulation.*

If the radius of both landmarks is unknown and different, by applying the method described in the previous point we obtain an "unbalanced" proportional triangulation (Fig. 4(b), in red color), caused by the disparity in the proportional calculation of the segment $b$ from the points $R$ and $H$.

Unknowing the radius of the observed landmarks, and considering that both radii can be different (or not), it can be asserted that the magnitude of one of them will be equal to the other one, multiplied by a given factor $k$; that is to say, $r_1 = r_2 \cdot k$. Maintaining the supposition of the fact that the radius $r_2$ possesses the magnitude 1, and since the magnitude of the segments $b$ of both triangulations must be identical in the reality, it is possible to calculate the mentioned factor $k$ (henceforth, *balancing factor*) in this way:

$$
\begin{aligned}
R_1'' &= \frac{k}{\sin(R\theta_1/2)} = R_1' \cdot k \quad ; \quad R_2' = \frac{1}{\sin(R\theta_2/2)} \\
H_1'' &= \frac{k}{\sin(H\theta_1/2)} = H_1' \cdot k \quad ; \quad H_2' = \frac{1}{\sin(H\theta_2/2)}
\end{aligned}
\tag{5}
$$

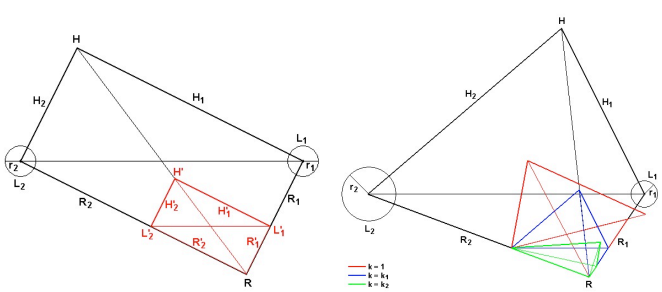
$$
\begin{aligned}
& b^2 = {H_1''}^2 + {H_2'}^2 - 2 \cdot H_1'' \cdot H_2' \cdot \cos H\alpha \quad ; \\
& b^2 = {R_1''}^2 + {R_2'}^2 - 2 \cdot R_1'' \cdot R_2' \cdot \cos R\alpha \quad ; \\
& 0 = {H_1''}^2 - {R_1''}^2 + 2 \cdot R_1'' \cdot R_2' \cdot \cos R\alpha \\
& \quad - 2 \cdot H_1'' \cdot H_2' \cdot \cos H\alpha + {H_2'}^2 - {R_2'}^2 \quad ; \\
& 0 = k^2 \cdot \left({H_1'}^2 - {R_1'}^2\right) \\
& \quad + k \cdot 2 \cdot \left(R_1' \cdot R_2' \cdot \cos R\alpha - H_1' \cdot H_2' \cdot \cos H\alpha\right) \\
& \quad + {H_2'}^2 - {R_2'}^2
\end{aligned}
\tag{6}
$$

Once calculated the balancing factor $k$, we can get a proportional triangulation balanced and equivalent to the real one, sufficient for the determination of the relative course to the point $H$ from the point $R$, through the resolution of the triangulation (3), and using $R_1''$, $R_2'$, $H_1''$ and $H_2'$ instead of $R_1$, $R_2$, $H_1$ and $H_2$, respectively ($R_1''$ and $H_1''$ correspond to $R_1'$ and $H_1'$, multiplied by the calculated factor $k$).

Nevertheless, note that the calculation of the balancing factor $k$ is accomplished through the resolution of a quadratic equation with two solutions, which will be denoted as $k_1$ (for the sum of the square root, in blue color in Fig. 4(b)) and $k_2$ (for the subtraction of the square root, in green color in Fig. 4(b)). Only one of the solutions is correct.

Unfortunately, we still have not found an effective criterion to discriminate which of both factors is the correct one for each couple of observations; but we have determined experimentally that: a) at most, only one of the factors is negative (in this case the correct one is the positive); and b) the variation of the correct factor in the vicinity of a given point $R$ is much less than that of the wrong factor, no mind the considered point and the real magnitude of the radii $r_1$ and $r_2$ (in fact, the correct factor is constant for each scene).

Considering the constancy of the correct balancing factor $k$ (henceforth, *certain*), this one can be determined depending on the calculations carried out in the recent past during navigation. For this purpose, one or two consecutive observations in movement (navigation stages) are enough, without significant loss of navigation efficiency (see discussion in V-A).




(a) (b)

Fig. 4. Proportional Triangulations (depicted in red color), increased and superposed to the real ones (in black color) at the observation point $R$: (a) With $r_1 = r_2$. (b) With $r_1 \neq r_2$, and showing the balanced proportional triangulations (the right one in blue and the wrong one in green).

### *III-C. Algorithm for homing through balanced proportional triangulation.*

Subsequently we reflect the pseudo-code of the algorithm for homing navigation through the proposed method:

**Algorithm 1**: Visual Homing by means of Balanced Proportional Triangulation

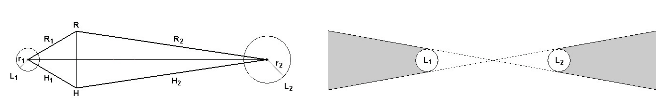
**Input**: OH = Observation accomplished in the home point.
**Output**: Upon ending the algorithm, the robot reaches the home point.
**Algorithm**:
Be: OR = Observation; k = float_number; Relative_Bearing = angle;

OR ← Observation(); //– Observation from current pose.
**While NOT**(Home_Reached(OH,OR)) **do**
**Calculate**: $H_1'$, $H_2'$, $R_1'$, $R_2'$; //– Equation (4).
**If NOT**(k_is_certain()) **then**
**Calculate**: $k_1$, $k_2$; //– Equation. (6).
$k$ ← Select_$k_1$_or_$k_2$(); //– According to previous stages.
**End_If**
$H_1''$ ← $H_1' * k$;
$R_1''$ ← $R_1' * k$;
**Calculate**: Relative_Bearing; //– Equation (3).
Move_Robot(Relative_Bearing);
OR ← Observation(); //– Repeat observation from current pose.
**End_While**
Stop_Robot(); //– Home reached.

### *III-D. Method exceptions.*

*III-D.1. Singular points:* For each home point of any scene, there are only two singular points in the space 2D where the application of the proposed method is not possible:

- Identity Point: it corresponds to the navigation stage when the observation accomplished from the point $R$ (current pose) is equivalent to the one which was carried out from the point $H$ (home); that is to say: $R\theta_1 = H\theta_1$, $R\theta_2 = H\theta_2$ and $R\alpha = H\alpha$. In this case, the home point has been reached and navigation must be stopped.
- Symmetrical Point: it corresponds to the navigation stage when the observation accomplished from the point $R$ (current pose) is symmetrical to the one which was carried out from the point $H$ (home), in relation to the axis formed by the centroids of the landmarks $L_1$ and $L_2$ (see Fig. 5(a)); that is to say: $R\theta_1 = H\theta_1$, $R\theta_2 = H\theta_2$ and $R\alpha \neq H\alpha$. In this case, the correct relative bearing is obtained by resolving the triangulation (3) through the situational assignment of the value 1 to the balancing factor $k$ (that is to say, considering $r_1 = r_2$).




(a) (b)

Fig. 5. (a) Symmetry between $H$ and $R$. (b) Dark zones (in gray color).

*III-D.2. Space of application:* The space of application of the proposed triangulation method corresponds to the whole of points of the space 2D from where both landmarks are thoroughly visible in a panoramic image, existing two dark zones (Fig. 5(b)) which depend on both the size of each landmark and the distance between them. Occasionally some of these two zones will be crossed during the homing navigation, being able to define avoidance or traffic strategies - even early detection strategies - (see discussion in V-C and V-D).

## IV. ACCOMPLISHED EXPERIMENTS.

The experiments with the proposed homing navigation method have been limited to its simulation in order to test its efficiency.

A visual module was built for that purpose (by means of a Java applet). It simulates a space of 8 x 6 m, obstacle-free and uncertainty-exempt, and with two landmarks mutually distinguishable and identifiable (see Fig. 6). This module permits to vary at convenience the size of each landmark as well as the positions of such landmarks, the home point and the initial location of the robot. It also permits to show the navigation path followed by the robot, as well as the calculations of each navigation stage.

Two types of experiments were made, corresponding to the homing navigation simulation of short and long range; in all the experiments, the position and size of each landmark were determined randomly. In all the accomplished simulations, the home point was reached by means of a rectilinear path and with a lower error margin than a hundredth part of millimeter ($10^{-5}$ m).

### *IV-A. Efficiency monitoring in proximity homing navigation.*

The objective of this group of experiments is to prove the efficiency of the proposed homing navigation method in environments where the starting point is located near the home point.

- Experiment 1: Random location of the robot and the home point. More than 500 test were carried out, placing randomly the robot as well as the home point on the simulation scene.

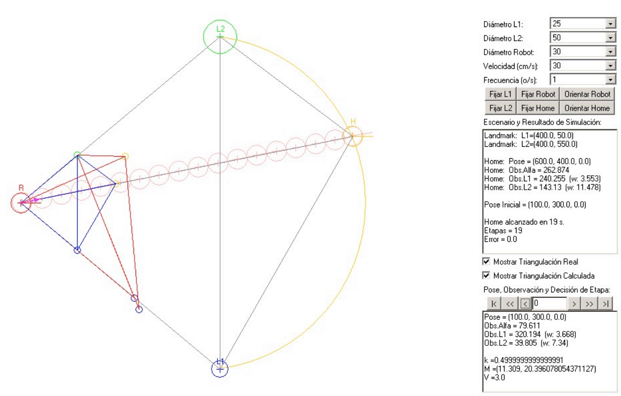



Fig. 6. Homing navigation simulation by means of the built applet.

- Experiment 2: Reaching a given home point from any starting point of the simulation scene. With the home point randomly placed, it was simulated the navigation from all the possible starting points of the simulation scene (8 x 6 m), discreeted each 3 cm (53,801 simulated paths).
- Experiment 3: Reaching any home point of the simulation scene from a given starting point (inverse experiment to the former one). With the robot randomly placed, it was simulated the navigation towards all the possible home points of the simulation scene (8 x 6 m), discreeted each 3 cm (53,801 simulated paths).

### *IV-B. Efficiency monitoring in long-range homing navigation.*

The objective of this experiment is to prove the efficiency of the proposed homing navigation method in environments where the starting point is located at a long distance from the home point.

- Experiment 4: Reaching a given home point from remote points of the simulation simulation scene. More than 50 tests were carried out, placing randomly the home point in the simulation scene (8 x 6 m), as well as the robot in a different distant point out of the simulation scene -8 Km away from the home point-.

## V. PROPOSED METHOD DISCUSSION.

The approach to homing navigation through the proposed method defers radically from other existing approaches, both equivalent [3] and non-equivalent (due to the use of the compass) [7] or [10], mainly because:

1. Topologically, it considers only two landmarks of the environment, instead of the minimum of three landmarks required in other approaches. This reduces the computational cost related to the detection and identification of landmarks; at the same time, it solves the landmark alignment problem in symmetrical environments.
2. Methodologically, it uses triangulation/trilateration to obtain the home bearing in each navigation stage, instead of vectorial motivation methods used in other approaches. This favors the obtainment of rectilinear navigation paths with no circumvolutions; at the same time, it solves the problem of discreeting navigation control magnitudes.

Furthermore, it provides the following advantages with regard to other approaches, due to the fact that it is a method that:

- It is valid for navigation throughout the 2D space, even in dark zones (see V-C).
- It is valid for any scene configuration, since it does not possess any restrictions for landmarks alignment.
- It minimizes the navigation time, since the calculated bearing generates a rectilinear path without circumvolutions.
- Due to its reactivity and robustness, it permits a simple integration with other navigation systems (e.g.: obstacle

avoidance), as well as its consideration for robots with a high level of motion uncertainty.
- It possesses a reduced and constant computational cost for each navigation stage.
- It simplifies the implementation, since it is based on the execution of a set of reduced operations, and it does not need any automatons for navigation control.

In spite of the efficiency of the proposed method for long-range navigation, whose experimental simulation is described in IV-B for an ideal environment of infinite visual radius, it is limited to the 2D space where the landmarks are mutually visible and distinguishable. Out of this boundary, highest-level navigation strategies will have to be implemented, in order to permit homing navigation throughout adjacent visibility sections of landmarks (as in [1] and [2]).

In the following paragraphs, we discuss various aspects concerning the definition of strategies so as to increase the efficiency and robustness of the proposed navigation method; we also discuss certain implications derived from the application of the method, as well as the parallelism which maintains with homing behaviors observed in animals.

### *V-A. Strategies for determination of the balancing factor (*k*).*

According to what was specified in III-B, for the certain determination of the factor $k$ during homing navigation, between one and two consecutive observations are enough, since:

1. If we obtain one of the factors $k_i$ as negative in the first observation, the positive will be the correct, and no movement with wrong bearing will be carried out.
2. If in the first observation both factors $k_i$ are positive, one of them will have to be chosen randomly, discriminating the right one in the second observation (the one with a lower variation). If in the first stage the right factor was not chosen, we will just have accomplished a movement with a wrong bearing.

Nevertheless, homing navigation is generally just the final phase of a far-reaching navigation plan, in which the robot will accomplish a previous navigation phase to the homing, which will move it away from the home point following a given path. Therefore, it is possible to determine the balancing factor $k$ during the previous navigation stages to the homing, and to link it to the observation accomplished at the home point, thus getting the homing navigation to be correctly directed from the first stage.

### *V-B. Strategies for robot speed control.*

The proposed navigation method rationalizes the robot speed control during the navigation, permitting to apply the maximum speed at each path stage, thus minimizing the navigation time, since:

1. The motion vector ($\overrightarrow{M}$) consists of a direction (relative) and a module, considered this as a scalar magnitude representing the distance to travel, but it is generally difficult to quantify and catalogue as linear measure [3]. The proposed navigation method not only calculates the direction of $\overrightarrow{M}$, but also the proportional distance to the home point ($d$ in (3) and Fig. 3), which can be assumed as a module of $\overrightarrow{M}$.
2. This proportional distance to the home point, calculated at each navigation stage and compared with the calculation of the previous stage, serves as a movement profit indicator between stages. A simple proportional speed calculation based on the profit between stages permits to assign the maximum speed to the current stage.

### *V-C. Strategies for the navigation in dark zones.*

It is possible to define various navigation strategies which permit to avoid and/or to cross the dark zones defined in III-D.2, so that the proposed navigation method will be applicable in the whole 2D space. For example, when the robot reaches one of the dark zones can adopt some of the following strategies:

- Traffic strategy: To cross the dark zone, it is enough to keep the last calculated bearing, since this course points out towards the home point.
- Avoidance strategy: To leave a dark zone, it is enough to turn the bearing $\pi$ radians (setback). Subsequently, we can fix the bearing towards the bisector of $\alpha$, until $\alpha$ reaches a magnitude near $\pi$ radians (axis of both landmarks), applying then again the proposed navigation method.

### *V-D. Implications derived: Proportional map and relative location.*

Once obtained the certain balancing factor $k$, the proposed triangulation method builds a proportional map of the environment at each navigation stage (on an indefinite scale, and representing exclusively the relative position of: the landmarks, the current pose and the home point). Therefore:

1. Implicitly, a relative location of the robot in the scene is taking place in a continuous way, with relation to the two landmarks observed.
2. With the certain balancing factor $k$ for a scene, the proportional distances from the home point to each landmark are known, as well as the proportional distance between both of them.
3. Considering different mutually adjacent scenes (sharing one of the landmarks and the home point), the separate obtained proportional maps can be integrated in one of a higher level.

Additionally, with the certain balancing factor $k$, and by resolving the corresponding Euclidean equations, we can determine the two dark zones of the scene in the proportional map of the environment before they are reached. This way the two dark zones can be avoided during navigation by means of some path planning strategy.

### *V-E. Parallelism with animal behavior studies.*

As it was indicated in I, there are various studies and experiments in Psycho-Biology about visual homing navigation observed in animals. Some of them, regarding bees and ants behavior, have been computational-modeled by proposing various strategies which reproduce plausibly the behaviors observed in these animals. However, they all agree to suggest a capacity for reference alignment concerning an external direction system (position of the sun, orientation of the land magnetic field); this external direction capacity reduces the parallelism with the proposed homing navigation method.

On the other hand, studies and experiments on learning of water rats [11] maintain certain parallelism with the proposed homing navigation method, since: 1) Water rats do not seem to have a biological compass for their external orientation [9]; 2) Without any absolute external references, they complete the homing navigation with a minimum of two landmarks [13]; and 3) They just use two landmarks to complete the homing, though there are some more in the scene [14].

In absence of a biological compass, and with only two landmarks of reference, water rats may be using some home navigation method equivalent to the proposed one; in this case, their method could consider the apparent size of the landmarks.

To determine the validity of the homing navigation method proposed as a plausible explanation for water rats homing behavior, a battery of experiments can be modeled to prove if these animals: a) Consider the size of the landmarks; b) Do not use stereoscopic vision to estimate the distance to the landmarks; and c) Do not use vectorial motivation methods towards the objective.

## VI. Conclusions and future work.

This document proposes a novel approach to the visual homing navigation, based on the calculation of the home bearing by means of triangulation/trilateration, starting from the observation in panoramic images of only two landmarks of the environment. For that purpose, it considers the angular difference observed between both landmarks and the apparent size of each one, and it does not use a compass or any range sensors.

The proposed approach, which maintains a certain parallelism with some given animal behaviors, has been simulated with sufficient intensity in an obstacle-free and exempt of uncertainty environment. As a result, this approach demonstrates to be complete (applicable throughout the 2D space), efficient (it generates rectilinear paths), reactive (it reconsiders the scene at each navigation stage) and powerful (with strategies to solve singular cases). At the same time it is simple (linear) and versatile (easily integratable with other navigation systems).

The new homing navigation method presented can be useful in environments which require great efficiency and navigation robustness and/or with a limited number of available landmarks (due to symmetry, alignment and/or excess of computational cost for their detection and identification).

Additionally, we suggest the possibility of using the proposed approach as a method for building proportional maps of the environment by means of their successive integration, due to the fact that the proposed method provides implicitly a proportional location system.

As projects and future research, we can emphasize the following ones:

- The implementation of the proposed approach in a robot.
- The simplification of the proposed approach by defining a direct calculation criterion of the balancing factor $k$, regardless of the former navigation stages.
- The study of the effectiveness of the proposed method with different sections of landmarks.
- The conceptual modeling (and the proposal to the competent scientific community) of psycho-biological experiments which could determine if the proposed homing navigation approach could explain the homing behavior observed in water rats in a plausible way.

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